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ESTIMATION OF THE SCALE PARAMETER
OF THE LOGNORMAL DISTRIBUTION
BY M ORDER STATISTICS

THESIS

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THESIS

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Preface

This thesis presents two relatively simple techniques for estimating the scale parameter σ of the lognormal probability density function through the use of order statistics. It has been assumed that the reader has a basic understanding of elementary statistics and references are provided for those readers desiring further reading about the lognormal distribution and order statistics.

I am very grateful to Dr. H. Leon Harter of the Aerospace Research Laboratories for suggesting the topic, acting as my thesis sponsor and making his valuable time available to me for necessary consultations. I am also extremely grateful and indebted to Professor A. H. Moore of the Mathematics Department for being my thesis advisor and suggesting and directing the method of approach in developing the m-order statistic estimator. Without his continual help, guidance and encouragement I would not have brought this study to a successful conclusion.

Finally, I wish to express my appreciation to my wife who has been very patient with me during some very trying periods.

Eugene R. Highfield

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Abstract

This thesis develops unbiased single order statistic and m-order-statistic estimators of the scale parameter σ of a truncated lognormal probability density function. In this development it is assumed that the location parameter μ is known and that if it is not zero, a transformed variable with a location parameter approximately zero can be obtained. The method of development of the single-order-statistic estimator was to consider the expected value of the i-th order statistic. The development of the m-order-statistic estimator utilizes the development of the single-order-statistic estimator and then considers the variance of the m-order-statistic estimator as a Lagrangian function which is minimized to obtain the necessary weighting factors. These weighting factors are then combined with the other coefficients to obtain the desired multipliers. All the multipliers used to obtain the estimators, the variance of the estimators and their relative efficiencies are presented in tabular form as appendices. The procedures to be followed in the proper use of these tables are contained in Chapter VI. The efficiency of the single order statistic is given relative to the m-order-statistic estimator which uses all the data available. In this respect, it was found that the best order statistic to use is the first one and that it produces efficiencies above 70% when the number of failures is less than half of the sample. The tabled multipliers and variances are given to five decimal places, the efficiencies to two decimal places and all numbers are accurate to within one unit in the last decimal place.

I. Introduction

The lognormal distribution, in its simplest form, may be defined as the distribution of a variate whose logarithm obeys the normal law of probability. Its history may be traced back to 1879 when D. McAlister set down explicitly, and in some detail, a theory of the lognormal distribution (Ref 1:2). Although little has been done in the theory of estimation for truncated or censored samples of a lognormal population, Sarhan and Greenberg (Ref 24) have worked in this area for the normal distribution using the method of least squares for estimators of μ and σ. Their work is somewhat similar to the work in this thesis in that both efforts utilize order statistics in the estimators. With the lognormal distribution, the data come in a natural ascending order whereas in the normal distribution the whole sample must be observed and then arranged in ascending order to obtain the order statistics.

Developments in the field of order statistics have occurred primarily in the last twenty years. The first two papers of real importance in the field were written by Mosteller (Ref 19) in 1946 and Wilks (Ref 28) in 1948. Much has been written since that time and a book edited by Sarhan and Greenberg (Ref 25), published in 1962, brings together many important works pertaining to the theory and application of order statistics.

The lognormal distribution is treated in this thesis with estimating tecnniques being derived to estimate the scale parameter σ. The development of the one-order-statistic estimator is similar to

that of Quayle (Ref 20), however the development of the m-order-statistic estimator is completely new and uses the Lagrangian method to minimize the variance of the estimator. In deriving the estimator it was assumed that the location parameter μ was zero.

It was necessary to consider various sample sizes, n, since each order statistic has a different probability distribution for each sample size. In this thesis, samples of sizes up to twenty are considered. An IBM 7094 digital computer was used to compute all tables and complete information on the use of these tables is presented.

II. Lognormal Distribution

Properties

The lognormal distribution is a distribution in which the logarithm of a random variable is normally distributed and either natural or common logarithms may be used (Ref 17:122). Although very little is written on the lognormal distribution in the standard textbook on statistics, a very thorough coverage is provided by Aitchison and Brown (Ref 1). Their work covers the history, general properties, estimation problems, examples of uses of the distribution and also describes variations of the lognormal distribution. For the purposes of this thesis, the lognormal distribution will be considered to be the two-parameter, positively skewed lognormal probability density distribution which is expressed as

$$f(t,\mu,\sigma) = f(t) = \frac{1}{(2\pi)^{1/2}\,\sigma\, t}\, e^{-\frac{1}{2}\left(\frac{\ln t - \mu}{\sigma}\right)^2} \qquad (2.1)$$

for $t > 0$, $\sigma > 0$ and $-\infty < \mu < \infty$

where ln t denotes the natural logarithm of t. The parameters μ and σ are the mean and standard deviation of ln t which is distributed normally. For this study, it is assumed that the mean is zero or that if it is not, it is known and the data adjusted. This is not an unreasonable assumption and if the mean is not known, it may be estimated (Ref 23). The moments of the lognormal distribution are defined by

$$E[t^r] = e^{r\mu + \frac{1}{2}(r\sigma)^2} \tag{2.2}$$

It can be seen that the mean α and variance β^2 are therefore given by

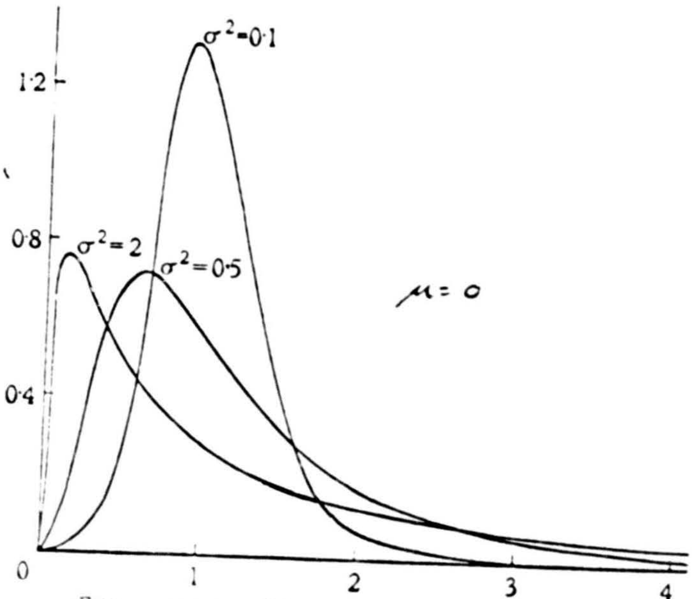
$$\alpha = e^{\mu + \frac{1}{2}\sigma^2} \tag{2.3}$$

$$\beta^2 = e^{2\mu + \sigma^2}(e^{\sigma^2} - 1) \tag{2.4}$$

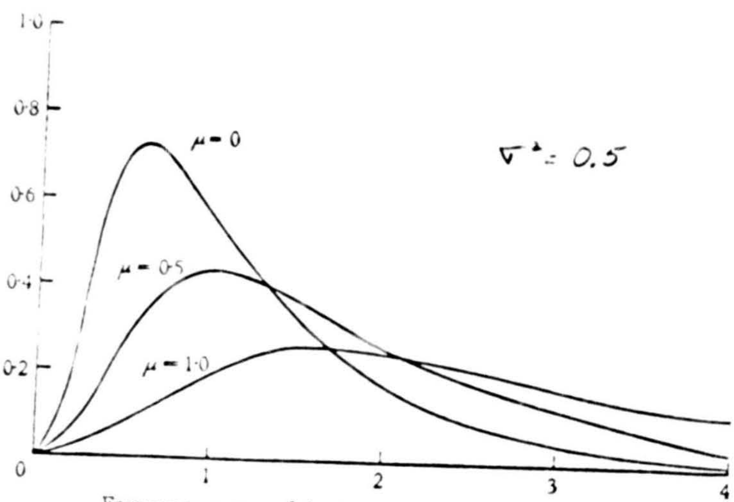
Figure 1 shows the lognormal probability density function for various values of μ and σ^2. It will be noted that the greater the value of σ^2, the greater is the skewness and the further to the right is the mean α. Another interesting feature of the lognormal distribution is that it has multiplicative reproductive properties whereas the normal distribution has additive reproductive properties.

Application

The lognormal distribution finds application in a wide variety of fields, such as physics, engineering, economics, biology, astronomy and sociology. It has been used to represent the distribution of incomes, of household size, of particle size, of body weight and of results of endurance tests (Ref 1:100-106). It has also been used to describe survival time of bacteria in disinfectants and the number of plankton organisms caught in a net (Ref 4).
In engineering, it represents the downtime of complex electronic systems (Ref 2:3), of servicing times in congestion problems (Ref 14) and also the repair time of airborne radar equipment (Ref 22:110).



Frequency curves of the lognormal distribution for three values of σ^2.



Frequency curves of the lognormal distribution for three values of μ.

Fig. 1

A rough test of whether a distribution is lognormal may be made by plotting the distribution on logarithmic probability paper on which the curve should approximate a straight line.

III. Order Statistics

General

Since this thesis is based on the concepts and properties of order statistics, their notations and definitions will be introduced. First, consider the elements y_1, y_2, ... y_n of a sample drawn from a continuous population $f(x)$ which are rearranged in ascending order of magnitude and then denoted $x_{1,n}$, $x_{2,n}$, ... $x_{n,n}$. This permutation of the original observations is referred to as the "order statistics." Various other quantities based on the order are also thought of as order statistics. In particular, any component $x_{i,n}$ is called the i-th order statistic. It is possible to derive the distribution of the individual order statistics or the joint distribution of several of them, however, for this paper, they will be stated and if the reader is interested in the detail, he is referred to Wilks (Ref 28) and Sarhan and Greenberg (Ref 25).

Probability Density Function

The probability density function of the i-th order statistic in a sample of size n from some general population is given by

$$g(x_{in}) = \frac{n!}{(i-1)!(n-i)!} F(x_{in})^{i-1} \left[1 - F(x_{in})\right]^{n-i} f(x_{in}) \qquad (3.1)$$

where $f(x_{in})$ and $F(x_{in})$ are the probability density function and the cumulative distribution function, respectively, of x_{in}. The mean value

of the i-th order statistic is

$$E[X_{in}] = \frac{n!}{(i-1)!(n-i)!}\int_{-\infty}^{\infty} X_{in}\, F(X_{in})^{i-1}\left[1-F(X_{in})\right]^{n-i} f(X_{in})\, dX_{in} \tag{3.2}$$

while the variance of the i-th order statistic is

$$VAR[X_{in}] = \frac{n!}{(i-1)!(n-i)!}\int_{-\infty}^{\infty} X_{in}^{2}\, F(X_{in})^{i-1}\left[1-F(X_{in})\right]^{n-i} dF(X_{in}) - E^{2}[X_{in}] \tag{3.3}$$

Also, the covariance of the i-th and j-th ($i < j$) order statistics is given by

$$COVAR[X_{in}, X_{jn}] = \frac{n!}{(i-1)!(j-i-1)!(n-j)!}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} X_{in} X_{jn}\, F(X_{in})^{i-1} \cdot \left[F(X_{jn}) - F(X_{in})\right]^{j-i-1}\left[1-F(X_{jn})\right]^{n-j} dF(X_{in})\, dF(X_{jn}) - E[X_{in}]\cdot E[X_{jn}] \tag{3.4}$$

The values for the above equations can be calculated after specifying the population.

<u>Lognormal Distribution</u>

Consider the r-th moment about the origin of the i-th order statistic t_{in} from a lognormal population which is expressed as

$$E[t_{in}^{r}] = \frac{n!}{(i-1)!(n-i)!}\int_{0}^{\infty} F(t_{in})^{i-1}\left[1-F(t_{in})\right]^{n-i} \cdot \frac{t_{in}^{r}\, e^{-\frac{1}{2}\left(\frac{\ln t_{in} - \mu}{\sigma}\right)^{2}}}{(2\pi)^{1/2}\, \sigma\, t_{in}}\, dt_{in} \tag{3.5}$$

now if a substitution of variables is made where

$$X_{in} = \frac{\ln t_{in} - \mu}{\sigma} \quad \text{and} \quad \sigma\, d X_{in} = \frac{d t_{in}}{t_{in}}$$

then

$$E\left[t_{in}^{r}\right] = \frac{n!}{(i-1)!(n-i)!} \int_{-\infty}^{\infty} F(X_{in})^{i-1} \left[1-F(X_{in})\right]^{n-i} \frac{e^{r\sigma X_{in} + r\mu} \cdot e^{-\frac{1}{2} X_{in}^{2}}}{(2\pi)^{1/2}} \, d X_{in} \quad (3.6)$$

$$= \frac{n! e^{r\mu} \cdot e^{\frac{(r\sigma)^2}{2}}}{(i-1)!(n-i)!} \int_{-\infty}^{\infty} F(X_{in})^{i-1} \left[1-F(X_{in})\right]^{n-i} \frac{e^{-\frac{1}{2}(X_{in} - r\sigma)^2}}{(2\pi)^{1/2}} \, d X_{in} \quad (3.7)$$

For ease of notation and simplicity, let

$$A_{r,i,n} = \frac{n!}{(i-1)!(n-i)!} \int_{-\infty}^{\infty} F(X_{in})^{i-1} \left[1-F(X_{in})\right]^{n-i} \frac{e^{-\frac{1}{2}(X_{in} - r\sigma)^2}}{(2\pi)^{1/2}} \, d X_{in} \quad (3.8)$$

then Eq (3.7) simplifies to

$$E\left[t_{in}^{r}\right] = e^{r\mu + \frac{(r\sigma)^2}{2}} \cdot A_{r,i,n} \qquad (3.9)$$

Now it will be recalled from chapter II that

$$E\left[t^{r}\right] = e^{r\mu + \frac{1}{2}(r\sigma)^2} \qquad (2.2)$$

and therefore Eq (3.9) may be restated as

$$E\left[t_{in}^{r}\right] = E\left[t^{r}\right] \cdot A_{r,i,n} \qquad (3.10)$$

Now if r is set equal to one, Eq (3.10) becomes

$$E\left[t_{in}\right] = E\left[t\right] \cdot A_{1,i,n} \qquad (3.11)$$

Although the above could be used as a method of obtaining the moments of the order statistics, it is not in a linear form which could be used to estimate the parameter σ. If, however, $E[\ln t_{in}]$ is considered rather than $E[t_{in}]$, it is possible to develop a linear relationship suitable to be used in estimation techniques. It will be seen however, that one is really looking at a normal density function when this is done. Consider then

$$E\left[(\ln t_{in})^r\right] = \frac{n!}{(i-1)!(n-i)!}\int_0^\infty F(t_{in})^{i-1}\left[1-F(X_{in})\right]^{n-i}(\ln t_{in})^r \cdot \frac{e^{-\frac{1}{2}\left(\frac{\ln t_{in}-\mu}{\sigma}\right)^2}}{(2\pi)^{1/2}\,\sigma\, t_{in}}\,dt_{in} \tag{3.12}$$

Once again, a substitution of variables is made by letting

$$X_{in} = \frac{\ln t_{in}-\mu}{\sigma} \qquad \text{then} \qquad \sigma\, dX_{in} = \frac{dt_{in}}{t_{in}}$$

and $\ln t_{in} = \sigma X_{in} + \mu$

then

$$E\left[(\ln t_{in})^r\right] = \frac{n!}{(i-1)!(n-i)!}\int_{-\infty}^{\infty} F(X_{in})^{i-1}\left[1-F(X_{in})\right]^{n-i}(\sigma X_{in}+\mu)^r \cdot \frac{e^{-\frac{1}{2}X_{in}^2}}{(2\pi)^{1/2}}\,dX_{in} \tag{3.13}$$

It should be noted that x_{in} is a standard normal variate. For

simplicity here and in later use, let

$$C_{in} = \frac{n!}{(i-1)!(n-i)!} \int_{-\infty}^{\infty} F(X_{in})^{i-1} \left[1 - F(X_{in})\right]^{n-i} \frac{X_{in}\, e^{-\frac{1}{2}X_{in}^2}}{(2\pi)^{1/2}} dX_{in} \tag{3.14}$$

At this time Eq (3.14) should be recognized as the expected value of the i-th order statistic of a standard normal population. Now for r=1, Eq (3.13) becomes

$$E\left[\ln t_{in}\right] = \sigma C_{in} + \mu \int_{-\infty}^{\infty} F(X_{in})^{i-1} \left[1 - F(X_{in})\right]^{n-i} \frac{e^{-\frac{1}{2}X_{in}^2}}{(2\pi)^{1/2}} dX_{in} \tag{3.15}$$

where it can be seen that the integral in the second term is the Beta integral and therefore allows further simplification of Eq (3.15) to

$$E\left[\ln t_{in}\right] = \sigma C_{in} + \mu B(i, n-i+1) \tag{3.16}$$

This is rather interesting and may be of some value for further study and use in some iterative techniques. However, this form does not lend itself to a simple linear estimator of σ unless it is assumed that the mean is zero. With that assumption, Eq (3.16) becomes

$$E\left[\ln t_{in}\right] = \sigma C_{in} = \sigma E\left[X_{in}\right] \tag{3.17}$$

Similarly it can be shown that

$$VAR\left[\ln t_{in}\right] = \sigma^2 VAR\left[X_{in}\right] \tag{3.18}$$

and that

$$COVAR\left[\ln t_{in}, \ln t_{jn}\right] = \sigma^2 COVAR\left[X_{in}, X_{jn}\right] \tag{3.19}$$

J. B. Rosser tabulated, up to nineteen decimal places, the expected values of the i-th order statistics of a standardized normal population (Ref 27:412). Following this, D. Teichroew calculated the values of the product of the i-th and j-th order statistics in samples of size ≤ 20 and published his and a ten decimal place version of Rosser's work (Ref 27). Using these tables, Sarhan and Greenberg calculated and published the variances and covariances for order statistics of sample sizes ≤ 20 to ten decimal places (Ref 24). For the purposes of this paper, only eight decimal places were used.

IV. One-Order-Statistic Estimator

Introduction

An estimator based on one order statistic has the advantage of being extremely simple to use while providing a fairly good indication of the parameter being estimated. Also this type estimator should be of considerable benefit where it is desired to determine the parameter σ early in the testing cycle so that early design changes can be initiated if necessary. In this thesis, the efficiency of the single-order-statistic estimator is determined relative to an estimator which is based on all the data available. That is, the efficiency is relative to an m-order-statistic estimator as found in the next chapter and tabled in Appendix C. It will be noted in column 4, Table 1, Appendix A, that the minimum variance, single-order-statistic estimator is that estimator based on the first or last possible failure. Table 2, Appendix B, shows the efficiency of using an estimator based on the first failure.

Derivation of the Estimator

It is desirable to obtain a linear unbiased estimator of σ based on the i-th order statistic. In the case of the lognormal, the logarithm of the i-th order statistic is used. Recall that

$$E\left[\ln t_{in}\right] = \sigma C_{in} \tag{3.27}$$

then it can be said that the estimator of σ is

$$\bar{\sigma} = C_{in}^{-1} \cdot E\left[\ln \hat{t}_{in}\right] \tag{4.1}$$

where $E\left[\ln t_{in}\right]$ may be replaced by the actual $\ln t_{in}$ since the maximum likelihood estimator of $E\left[\ln t_{in}\right]$ is $\ln t_{in}$ itself. We then have

$$\bar{\sigma} = C_{in}^{-1} \cdot \left[\ln t_{in}\right] \tag{4.2}$$

where C_{in}^{-1} is tabulated in column 3 of Table 1 which is Appendix A of this paper, and is hereafter referred to as the "multiplier." For ease of notation in the next chapter, let

$$M_{in} = \text{multiplier} = C_{in}^{-1} \tag{4.3}$$

Variance of the Estimator

Recall from elementary statistics (Ref 5:71) that if

$$y = ax + b \tag{4.4}$$

then the variance of y may be expressed by

$$VAR\left[y\right] = a^2\, VAR\left[x\right] \tag{4.5}$$

which can be extended to the estimator and therefore the variance of the estimator may be given by

$$VAR\left[\bar{\sigma}\right] = M_{in}^2\, VAR\left[\ln t_{in}\right] \tag{4.6}$$

and then substituting Eq (3.18) into Eq (4.6), we have

$$VAR\left[\bar{\sigma}\right] = M_{in}^2\, \sigma^2\, VAR\left[X_{in}\right] \tag{4.7}$$

and therefore

$$VAR[\bar{v}]/v^2 = M_{in}^{\circ\,2}\, VAR[X_{in}] \tag{4.8}$$

By substituting Eqs (4.3) and (3.14) into the equation above, we see that

$$VAR[\bar{v}]/v^2 = \frac{VAR[X_{in}]}{E^2[X_{in}]} \tag{4.9}$$

where both the numerator and the denominator have been tabulated for n ≤ 20 (Ref 25:193,200-205) and therefore Eq (4.9) can be readily evaluated; this has been done and the results have been tabulated in column 4 of Table 1, Appendix A.

Efficiency of the Estimator

The relative efficiency of the estimator is based on the variance of the m-order-statistic estimator of the next chapter which uses all the available data and is a minimum variance estimator. The efficiency of the single-order-statistic estimator is therefore given as

$$EFF = \frac{VAR[\bar{v}_m]}{VAR[\bar{v}_s]} \cdot 100 \tag{4.10}$$

where $\bar{v}_m$ is the m-order-statistic estimator and $\bar{v}_s$ is the single-order-statistic estimator.

V. M-Order-Statistic Estimator

Introduction

While a single order statistic estimator is extremely simple to use, it is not too efficient for the normal and lognormal distributions because the variance of the estimator gets very large. Therefore, it is desirable to have an estimator which may be used when speed and simplicity are not as essential as efficiency. The estimator derived here will be linear and unbiased as well as one which uses all the available data.

Derivation of the Estimator

Consider the linear relationship

$$\bar{V} = \frac{k_1 M_{1n} \ln t_{1n} + k_2 M_{2n} \ln t_{2n} + \ldots + k_m M_{mn} \ln t_{mn}}{m} \qquad (5.1)$$

where n is the number of items on test, m is the failure at which the test is terminated (or truncated), the k_i are coefficients such that

$$\sum_{i=1}^{m} k_i = m \qquad (5.2)$$

and

$$M_{in} = \left(E\left[\ln t_{in}\right]\right)^{-1} \qquad (5.3)$$

Once the k_i are determined, then K_{in}, where

$$K_{in} = \frac{k_i M_{in}}{m} \qquad (5.4)$$

can be tabulated (see Table 3) and Eq (5.1) becomes

$$\bar{\sigma} = K_{1n} \ln t_{1n} + K_{2n} \ln t_{2n} + \cdots + K_{mn} \ln t_{mn} \tag{5.5}$$

which provides a linear estimate for σ of a normal distribution with a mean of zero. This estimator is unbiased and the proof follows. From Eq (5.1), the expected value is determined by

$$E[\bar{\sigma}] = \frac{1}{m} E\left[\sum_{i=1}^{m} k_i M_{in} \ln t_{in}\right] \tag{5.6}$$

and by combining Eq (3.17) and Eq (4.3)

$$E[M_{in} \ln t_{in}] = \sigma \tag{5.7}$$

therefore

$$E[\bar{\sigma}] = \frac{1}{m} E\left[\sum_{i=1}^{m} k_i \sigma\right] \tag{5.8}$$

but with Eq (5.2) the above equation becomes

$$E[\bar{\sigma}] = \frac{1}{m}(m\sigma) = \sigma \tag{5.9}$$

Since the estimator is linear, unbiased, and uses all available data, the k_i are now calculated so that the estimator will have a minimum variance.

Variance of the Estimator

The variance of the estimator given by Eq (5.1) may be written as

$$VAR[\bar{\sigma}] = \frac{k_1^2 M_{1n}^2}{m^2} VAR[\ln t_{1n}] + \cdots + \frac{k_m^2 M_{mn}^2}{m^2} VAR[\ln t_{mn}]$$
$$+ 2\sum_{j=2}^{m} \sum_{i=1}^{j-1} \frac{k_i M_{in} k_j M_{jn}}{m^2} Cov[\ln t_{in}, \ln t_{jn}] \tag{5.10}$$

which may be simplified and put in terms of the standard normally distributed variate x_{in} by utilizing Eqs (3.18) and (3.19), so that Eq (5.10) becomes

$$VAR[\tilde{\sigma}]/\sigma^2 = \frac{k_1^2 M_{1n}^2}{m^2} VAR[X_{1n}] + \cdots + \frac{k_m^2 M_{mn}^2}{m^2} VAR(X_{mn})$$

$$+ 2\sum_{j=2}^{m} \sum_{i=1}^{j-1} \frac{k_i M_{in} k_j M_{jn}}{m^2} COVAR[X_{in}, X_{jn}] \quad (5.11)$$

The variances and covariances of order statistics from a standard normal population have been tabulated by Teichroew (Ref 25:193, 200-205) for $n \leq 20$. Although the results are accurate to 20 decimal places, only the first 8 places have been used in this study. With the values just mentioned, the only unknowns in Eq (5.11) are the k_i. In order to select the k_i so as to minimize the variance of the estimator, let $Var[\tilde{\sigma}]/\sigma^2$ be a Lagrangian function subject to the constraint that

$$\sum_{i=1}^{m} k_i = m \quad (5.2)$$

and solve for the k_i by the standard Lagrangian method as shown in the next paragraph.

<u>Illustrative Example</u>

For illustrative purposes, consider the case where a sample of size n is on test and truncation of testing is made after two failures. That is, if m=2, then Eq (5.1) becomes

$$\tilde{\sigma} = \frac{k_1 M_{1n} \ln t_{1n} + k_2 M_{2n} \ln t_{2n}}{2} \quad (5.12)$$

and Eq (5.11), the Lagrangian function, becomes

$$\mathcal{V} = VAR[\bar{v}]/v^2 = \frac{k_1^2 M_{1n}^2}{4} VAR[X_{1n}] + \frac{k_2^2 M_{2n}^2}{4} VAR[X_{2n}]$$
$$+ 2\frac{k_1 M_{1n} k_2 M_{2n}}{4} COVAR[X_{1n}, X_{2n}] \quad (5.13)$$

Then, taking partial derivatives of $\mathcal{V}$ with respect to the variables k_i, we have

$$\frac{\partial \mathcal{V}}{\partial k_1} = \frac{k_1 M_{1n}^2}{2} VAR[X_{1n}] + \frac{k_2 M_{1n} M_{2n}}{2} COVAR[X_{1n}, X_{2n}] = 0 \quad (5.14)$$

$$\frac{\partial \mathcal{V}}{\partial k_2} = \frac{k_2 M_{2n}^2}{2} VAR[X_{2n}] + \frac{k_1 M_{1n} M_{2n}}{2} COVAR[X_{1n}, X_{2n}] = 0 \quad (5.15)$$

$$k_1 + k_2 = 2 \quad (5.16)$$

Now solving these equations simultaneously, one finds that

$$k_1 = \frac{2\left(M_{2n}^2 VAR[X_{2n}] - M_{1n} M_{2n} COVAR[X_{1n}, X_{2n}]\right)}{M_{1n}^2 VAR[X_{1n}] + M_{2n}^2 VAR[X_{2n}] - 2 M_{1n} M_{2n} COVAR[X_{1n}, X_{2n}]} \quad (5.17)$$

and

$$k_2 = \frac{2}{1 + \left(\frac{M_{2n}^2 VAR[X_{2n}] - M_{1n} M_{2n} COVAR[X_{1n}, X_{2n}]}{M_{1n}^2 VAR[X_{1n}] - M_{1n} M_{2n} COVAR[X_{1n}, X_{2n}]}\right)} \quad (5.18)$$

The results are then used in Eq (5.4) and then the K_{in} are utilized in Eq (5.5) to obtain the appropriate $\bar{v}$.

The K_{in} obtained by the above techniques have been tabulated

in Table 3 in columns 3 through 7 for the appropriate n and m. Further, the k_i were used in Eq (5.10) and then $Var [\bar{\sigma}] / \sigma^2$ was tabulated in column 8 of Table 3.

VI. Use of Tables

General

Before any of the tables are used, it must be determined that the applicable distribution is the lognormal (see Chapter II). These tables are also applicable to the normal distribution but all the sample data would be needed in that case and application would be similar to that used by Sarhan and Greenberg (Ref 24). If μ is not zero, it can be estimated (Ref 24) and a transformed variable derived which has a mean which is approximately zero.

Table 1 One-Order-Statistic Estimator

This table will not normally be used since the efficiency obtained by using the time of the first failure is much higher than that of estimators based on other order statistics. The difference in efficiency may be seen by comparing the efficiencies of Table 1 with those of Table 2. The steps to follow in using this table are as follows:

1. Locate the page of Table 1 which contains the sample size n in column (1).
2. Locate in column (2) the number of the failure at which it is desired to obtain an estimate of σ.
3. Estimate the scale parameter σ by multiplying the logarithm of $t_{n,m}$ by the value found in column (3). The value in column (3) is called the multiplier.

4. The variance of this estimator is found by multiplying the square of the true value of the scale parameter ∇ by the factor found in column (5).

5. The relative efficiency of this estimator as compared with an m-order statistic estimator is found in column (6).

Table 2 One-Order-Statistic Estimator

Since the first failure time provides an estimator which has the smallest variance and highest efficiency relative to the data collected among the one order statistic estimators, this table was prepared for its simplicity and so that the efficiency of the estimator could be seen. This is the table that should be used when using only one order statistic for estimation of ∇. The procedure is as follows:

1. Locate the page of Table 2 which contains the sample size n in column (1).

2. Locate the line with a one in column (2) for that sample size.

3. Estimate the scale parameter ∇ by multiplying the logarithm of $t_{1,n}$ by the multiplier in column (3).

4. The variance of this estimator is found by multiplying the square of the true value of the scale parameter ∇ by the factor found in column (4).

5. The relative efficiency of this estimator is found by locating the total failures under column (2) and then reading the relative efficiency which is on the same line

in column (5).

Table 3 M-Order-Statistic Estimator

The m-order statistic estimator provides an estimator which has minimum variance, unbiased and sufficient and should be used whenever possible to give a more accurate estimate of σ. The steps in using the table are as follows:

1. Locate the page of Table 3 which contains the sample size n in column (1).
2. Locate the line within that sample size which indicates the number of failures observed.
3. Estimate the scale parameter σ by multiplying the logarithm of each t_{nm} by the appropriate multiplier in columns (3) through (7). Multiply the time of first failure by first line K1, second failure time by K2, and continue. When the number of failures exceeds five, multiply the time of the sixth failure by the K1 on the second line, seventh failure time by the K2 on the second line, etc.
4. The variance of this estimator is found by multiplying the square of the true value of the scale parameter σ by the factor found in column (8) and on the same line as the last multiplier for this estimator.

VII. Conclusions and Recommendations

Conclusions

The necessary background theory on order statistics and the lognormal distribution have been presented in this thesis to allow estimation of the scale parameter σ of the lognormal pdf. Estimation techniques were developed for utilizing one order statistic or all available order statistics. Although the times-to-failure for the sample come in an ordered ascending sequence, a very long period of time could pass between successive failures. These hard to realize failures are not required by the one order statistic estimation techniques since it was found that the use of the first failure time, even though as many as half of the sample failed, provided estimators with relative efficiencies above 70% (See Table 2). The m-order-statistic estimator found in this thesis is a minimum variance estimator. The variances of the estimators were compared with those of the estimators determined by the method of least squares by Sarhan and Greenberg (Ref 21) and in each case, the variance of the m-order statistic estimator was equal to or less than the other comparable variance. In the case where the full sample fails, the multipliers are also equal to those determined by the method of least squares. This provides a good check on the accuracy of the multipliers of this thesis since they were derived by a different method.

It should be pointed out that, while the estimates of σ

obtained in this thesis are unbiased, estimates of α and β^2 obtained by substituting estimates of σ in Eqs (2.3) and (2.4) would be biased. In general, functions of unbiased estimators of parameters are not unbiased estimators of the corresponding functions of the parameters. One advantage of the method of maximum likelihood, now under study by Moore and Harter (Ref 12), is that functions of maximum likelihood estimates of parameters are maximum likelihood estimators of the corresponding functions of the parameters.

Although the theory developed here contains some rather involved mathematical expressions, the tables for the application of the theory are extremely simple to use.

Recommendations

Recommendations for further study are the extension of Table 3 to higher sample sizes, a rework of Table 3 to allow censoring much the same as was done by Sarhan and Greenberg (Ref 25:206-269) and finally, investigation of the possible use of equation (3.16) for parameter estimation by iterative techniques.

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APPENDIX A

TABLE 1

An Unbiased One-Order-Statistic Estimator
of the Parameter
of a 1-Parameter Lognormal Population

TABLE 1

AN UNBIASED ONE ORDER STATISTIC ESTIMATOR OF THE PARAMETER σ OF A 1 - PARAMETER LOG-NORMAL POPULATION

(1) N	(2) M	(3) MULTIPLIER	(4) VAR OF EST/σ^2	(5) EFF (%)
2	1	-1.772454	2.141593	100.00
2	2	1.772454	2.141593	26.65
3	1	-1.181636	0.781164	100.00
3	3	1.181636	0.781164	35.27
4	1	-0.971463	0.464051	100.00
4	2	-3.366874	4.086065	10.40
4	3	3.366874	4.086065	7.01
4	4	0.971463	0.464051	38.80
5	1	-0.859871	0.330897	100.00
5	2	-2.020125	1.271279	25.70
5	4	2.020125	1.271279	14.93
5	5	0.859871	0.330897	40.29
6	1	-0.789137	0.259014	100.00
6	2	-1.558227	0.678835	38.15
6	3	-4.961626	6.061194	3.84
6	4	4.961626	6.061194	3.06
6	5	1.558227	0.678835	20.72
6	6	0.789137	0.259014	40.81
7	1	-0.739547	0.214352	100.00
7	2	-1.320351	0.447569	47.48
7	3	-2.835215	1.766219	11.43
7	5	2.835215	1.766219	7.97
7	6	1.320351	0.447569	24.85
7	7	0.739547	0.214352	40.82
8	1	-0.702444	0.183998	100.00
8	2	-1.173399	0.329623	54.46
8	3	-2.114959	0.898049	19.56
8	4	-6.556758	8.047336	1.98
8	5	6.556758	8.047336	1.69
8	6	2.114959	0.898049	12.54
8	7	1.173399	0.329623	27.85
8	8	0.702444	0.183998	40.55
9	1	-0.673395	0.162046	100.00
9	2	-1.072619	0.259667	59.80
9	3	-1.748341	0.569715	27.06

TABLE 1

AN UNBIASED ONE ORDER STATISTIC ESTIMATOR OF THE PARAMETER σ OF A 1 - PARAMETER LOG-NORMAL POPULATION

(1) N	(2) M	(3) MULTIPLIER	(4) VAR OF EST/σ^2	(5) EFF(%)
9	4	-3.642643	2.263120	6.41
9	6	3.642643	2.263120	4.91
9	7	1.748341	0.569715	16.40
9	8	1.072619	0.259667	30.05
9	9	0.673395	0.162046	40.12
10	1	-0.649877	0.145430	100.00
10	2	-0.998645	0.212943	63.97
10	3	-1.524253	0.406593	33.62
10	4	-2.661 40	1.118555	11.78
10	5	-8.152102	10.038553	1.21
10	6	8.152102	10.038553	1.07
10	7	2.661240	1.118555	8.33
10	8	1.524253	0.406593	19.59
10	9	0.998645	0.212943	31.69
10	10	0.649877	0.145430	39.60
11	1	-0.630344	0.132410	100.00
11	2	-0.941694	0.181967	67.31
11	3	-1.372044	0.311977	39.26
11	4	-2.164604	0.692243	17.30
11	5	-4.446601	2.761020	4.10
11	7	4.446601	2.761020	3.31
11	8	2.164604	0.692243	11.53
11	9	1.372044	0.311977	22.22
11	10	0.941694	0.181967	32.92
11	11	0.630344	0.132410	39.04
12	1	-0.613788	0.121925	100.00
12	2	-0.896272	0.158463	70.03
12	3	-1.261291	0.251321	44.08
12	4	-1.862742	0.485117	22.58
12	5	-3.202574	1.339637	7.85
12	6	-9.747569	12.032502	0.81
12	7	9.747569	12.032502	0.74
12	8	3.202574	1.339637	5.91
12	9	1.862742	0.485117	14.39
12	10	1.261291	0.251321	24.38

TABLE 1

AN UNBIASED ONE ORDER STATISTIC ESTIMATOR OF THE PARAMETER σ OF A 1 - PARAMETER LOG-NORMAL POPULATION

(1) N	(2) M	(3) MULTIPLIER	(4) VAR OF EST/σ^2	(5) EFF(%)
12	11	0.896272	0.158463	33.85
12	12	0.613788	0.121925	38.45
13	1	-0.599524	0.113294	100.00
13	2	-0.859050	0.140518	72.28
13	3	-1.176700	0.209620	48.21
13	4	-1.658787	0.365990	27.47
13	5	-2.575148	0.817321	11.96
13	6	-5.248691	3.259501	2.84
13	8	5.248691	3.259501	2.38
13	9	2.575148	0.817321	8.51
13	10	1.658787	0.365990	16.91
13	11	1.176700	0.209620	26.17
13	12	0.859050	0.140518	34.55
13	13	0.599524	0.113294	37.85
14	1	-0.587068	0.106059	100.00
14	2	-0.827882	0.126399	74.18
14	3	-1.109722	0.179433	51.77
14	4	-1.511113	0.290519	31.91
14	5	-2.195071	0.564233	16.13
14	6	-3.741156	1.561036	5.59
14	7	-11.343114	14.028088	0.58
14	8	11.343114	14.028088	0.54
14	9	3.741156	1.561036	4.40
14	10	2.195071	0.564233	10.97
14	11	1.511113	0.290519	19.11
14	12	1.109722	0.179433	27.67
14	13	0.827882	0.126399	35.07
14	14	0.587068	0.106059	37.26
15	1	-0.576066	0.099901	100.00
15	2	-0.801324	0.115017	75.80
15	3	-1.055198	0.154697	54.86
15	4	-1.398841	0.239180	35.93
15	5	-1.939108	0.320646	20.18
15	6	-2.982439	0.941678	8.74
15	7	-6.049661	3.758351	2.08

TABLE 1

AN UNBIASED ONE ORDER STATISTIC ESTIMATOR
OF THE PARAMETER σ OF A
1 - PARAMETER LOG-NORMAL POPULATION

(1) N	(2) M	(3) MULTIPLIER	(4) VAR OF EST/σ^2	(5) EFF(%)
15	9	6.049661	3.758351	1.79
15	10	2.982439	0.941678	6.52
15	11	1.939108	0.420646	13.24
15	12	1.398841	0.239180	21.02
15	13	1.055198	0.156697	28.91
15	14	0.801324	0.115017	35.45
15	15	0.576066	0.099901	36.67
16	1	-0.566254	0.094593	100.00
16	2	-0.778365	0.105657	77.20
16	3	-1.009824	0.139031	57.55
16	4	-1.310330	0.202371	39.54
16	5	-1.754357	0.330404	24.04
16	6	-2.523833	0.643637	12.06
16	7	-4.278046	1.782632	4.18
16	8	-12.938710	16.024722	0.44
16	9	12.938710	16.024722	0.41
16	10	4.278046	1.782632	3.40
16	11	2.523833	0.643637	8.62
16	12	1.754357	0.330404	15.30
16	13	1.310330	0.202371	22.69
16	14	1.009824	0.139031	29.96
16	15	0.778365	0.105657	35.72
16	16	0.566254	0.094593	36.09
17	1	-0.557432	0.089967	100.00
17	2	-0.758275	0.097829	78.42
17	3	-0.971382	0.124950	59.92
17	4	-1.238567	0.174892	42.78
17	5	-1.614315	0.269463	27.65
17	6	-2.215657	0.475609	15.41
17	7	-3.387689	1.066197	6.66
17	8	-6.849905	4.257450	1.59
17	10	6.849905	4.257450	1.40
17	11	3.387689	1.066197	5.15
17	12	2.215657	0.475609	10.62
17	13	1.614315	0.269463	17.16

TABLE 1

AN UNBIASED ONE ORDER STATISTIC ESTIMATOR
OF THE PARAMETER σ OF A
1 - PARAMETER LOG-NORMAL POPULATION

(1) N	(2) M	(3) MULTIPLIER	(4) VAR OF EST/σ^2	(5) EFF(%)
17	14	1.238567	0.174892	24.15
17	15	0.971382	0.124950	30.85
17	16	0.758275	0.097829	35.91
17	17	0.557432	0.089967	35.53
18	1	-0.549441	0.085895	100.00
18	2	-0.740514	0.091189	79.49
18	3	-0.938326	0.113491	62.02
18	4	-1.179071	0.153710	45.70
18	5	-1.504224	0.226062	31.02
18	6	-1.993694	0.370615	18.70
18	7	-2.850325	0.723203	9.34
18	8	-4.813818	2.004360	3.24
18	9	-14.534341	18.022076	0.34
18	10	14.534341	18.022076	0.32
18	11	4.813818	2.004360	2.70
18	12	2.850325	0.723203	6.94
18	13	1.993694	0.370615	12.49
18	14	1.504224	0.226062	18.84
18	15	1.179071	0.153710	25.43
18	16	0.938326	0.113491	31.59
18	17	0.740514	0.091189	36.03
18	18	0.549441	0.085895	34.99
19	1	-0.542158	0.082283	100.00
19	2	-0.724670	0.085486	80.44
19	3	-0.909543	0.103999	63.88
19	4	-1.128844	0.136953	48.34
19	5	-1.415205	0.193860	34.13
19	6	-1.825792	0.300090	21.87
19	7	-2.489778	0.530741	12.13
19	8	-3.791571	1.190823	5.24
19	9	-7.649653	4.756724	1.25
19	11	7.649653	4.756724	1.12
19	12	3.791571	1.190823	4.17
19	13	2.489778	0.530741	8.69
19	14	1.825792	0.300090	14.23

TABLE 1

AN UNBIASED ONE ORDER STATISTIC ESTIMATOR
OF THE PARAMETER σ OF A
1 - PARAMETER LOG-NORMAL POPULATION

(1) N	(2) M	(3) MULTIPLIER	(4) VAR OF EST/σ^2	(5) EFF(%)
19	15	1.415205	0.193860	20.34
19	16	1.128844	0.136953	26.56
19	17	0.909543	0.103999	32.22
19	18	0.724670	0.085486	36.09
19	19	0.542158	0.082283	34.46
20	1	-0.535482	0.079054	100.00
20	2	-0.710427	0.080538	81.29
20	3	-0.884214	0.096020	65.55
20	4	-1.085798	0.123409	50.73
20	5	-1.341592	0.169181	37.00
20	6	-1.694063	0.250111	24.89
20	7	-2.230491	0.411004	14.92
20	8	-3.175276	0.802870	7.44
20	9	-5.348813	2.226181	2.59
20	10	-16.129997	20.019939	0.27
20	11	16.129997	20.019939	0.26
20	12	5.348813	2.226181	2.20
20	13	3.175276	0.802870	5.70
20	14	2.230491	0.411004	10.37
20	15	1.694063	0.250111	15.83
20	16	1.341592	0.169181	21.70
20	17	1.085798	0.123409	27.55
20	18	0.884214	0.096020	32.76
20	19	0.710427	0.080538	36.10
20	20	0.535482	0.079054	33.95

APPENDIX B

TABLE 2

An Unbiased One-Order-Statistic
Estimator of the Parameter
of a 1-Parameter Lognormal Population

TABLE 2

AN UNBIASED ONE ORDER STATISTIC ESTIMATOR OF THE PARAMETER σ OF A 1 - PARAMETER LOG-NORMAL POPULATION

(1) N	(2) M	(3) MULTIPLIER	(4) VAR OF EST/σ^2	(5) EFF(%)
2	1	-1.772454	2.141593	100.00
2	2			26.65
3	1	-1.181636	0.781164	100.00
3	2			100.00
3	3			35.27
4	1	-0.971463	0.464051	100.00
4	2			91.58
4	3			61.70
4	4			38.80
5	1	-0.859871	0.330897	100.00
5	2			98.73
5	3			98.73
5	4			57.35
5	5			40.29
6	1	-0.789137	0.259014	100.00
6	2			99.99
6	3			89.81
6	4			71.63
6	5			54.30
6	6			40.81
7	1	-0.739547	0.214352	100.00
7	2			99.13
7	3			94.19
7	4			94.19
7	5			65.69
7	6			51.90
7	7			40.82
8	1	-0.702444	0.18399[illegible]	100.00
8	2			97.56
8	3			95.46
8	4			86.74
8	5			74.12
8	6			61.21
8	7			49.89
8	8			40.55

TABLE 2

AN UNBIASED ONE ORDER STATISTIC ESTIMATOR
OF THE PARAMETER σ OF A
1 - PARAMETER LOG-NORMAL POPULATION

(1) N	(2) M	(3) MULTIPLIER	(4) VAR OF EST/σ^2	(5) EFF(%)
9	1	-0.673395	0.162046	100.00
9	2			95.82
9	3			95.13
9	4			89.59
9	5			89.59
9	6			68.54
9	7			57.67
9	8			48.16
9	9			40.12
10	1	-0.649877	0.145430	100.00
10	2			94.10
10	3			93.99
10	4			90.64
10	5			83.43
10	6			73.97
10	7			64.05
10	8			54.78
10	9			46.62
10	10			39.60
11	1	-0.630344	0.132410	100.00
11	2			92.50
11	3			92.49
11	4			90.57
11	5			85.39
11	6			85.39
11	7			69.04
11	8			60.36
11	9			52.34
11	10			45.25
11	11			39.04
12	1	-0.613788	0.121925	100.00
12	2			91.01
12	3			90.85
12	4			89.84
12	5			86.22

TABLE 2

AN UNBIASED ONE ORDER STATISTIC ESTIMATOR
OF THE PARAMETER σ CF A
1 - PARAMETER LOG-NORMAL PCPULATION

(1) N	(2) M	(3) MULTIPLIER	(4) VAR OF EST/σ^2	(5) EFF(%)
12	6			80.21
12	7			72.78
12	8			64.90
12	9			57.26
12	10			50.25
12	11			44.00
12	12			38.45
13	1	-0.599524	0.113294	100.00
13	2			89.65
13	3			89.20
13	4			88.73
13	5			86.27
13	6			81.64
13	7			81.64
13	8			68.46
13	9			61.38
13	10			54.62
13	11			48.43
13	12			42.85
13	13			37.85
14	1	-0.587068	0.106059	100.00
14	2			88.41
14	3			87.58
14	4			87.42
14	5			85.79
14	6			82.29
14	7			77.21
14	8			71.16
14	9			64.73
14	10			58.36
14	11			52.34
14	12			46.81
14	13			41.79
14	14			37.26
15	1	-0.576066	0.099901	100.00

TABLE 2

AN UNBIASED ONE ORDER STATISTIC ESTIMATOR
OF THE PARAMETER σ OF A
1 - PARAMETER LOG-NORMAL POPULATION

(1) N	(2) M	(3) MULTIPLIER	(4) VAR OF EST/σ^2	(5) EFF(%)
15	2			87.27
15	3			86.04
15	4			86.02
15	5			84.99
15	6			82.37
15	7			78.28
15	8			78.28
15	9			67.38
15	10			61.48
15	11			55.74
15	12			50.33
15	13			45.35
15	14			40.81
15	15			36.67
16	1	-0.566254	0.094593	100.00
16	2			86.23
16	3			84.59
16	4			84.58
16	5			83.97
16	6			82.06
16	7			78.79
16	8			74.45
16	9			69.40
16	10			64.02
16	11			58.63
16	12			53.44
16	13			48.55
16	14			44.04
16	15			39.90
16	16			36.09
17	1	-0.557432	0.089967	100.00
17	2			85.28
17	3			83.22
17	4			83.17
17	5			82.83

TABLE 2

AN UNBIASED ONE ORDER STATISTIC ESTIMATOR
OF THE PARAMETER σ CF A
1 - PARAMETER LOG-NORMAL PCPULATION

(1) N	(2) M	(3) MULTIPLIER	(4) VAR OF EST/σ^2	(5) EFF(%)
17	6			81.46
17	7			78.89
17	8			75.27
17	9			75.27
17	10			66.C5
17	11			61.C6
17	12			56.12
17	13			51.40
17	14			46.96
17	15			42.84
17	16			39.C5
17	17			35.53
18	1	-0.549441	0.085895	100.C0
18	2			84.39
18	3			81.94
18	4			81.79
18	5			81.63
18	6			8C.67
18	7			78.67
18	8			75.68
18	9			71.90
18	10			67.61
18	11			63.C4
18	12			58.41
18	13			53.89
18	14			49.57
18	15			45.51
18	16			41.74
18	17			38.25
18	18			34.99
19	1	-0.542158	0.082283	100.00
19	2			83.58
19	3			80.74
19	4			8C.46
19	5			80.40

TABLE 2

AN UNBIASED ONE ORDER STATISTIC ESTIMATOR
OF THE PARAMETER σ CF A
1 - PARAMETER LOG-NORMAL PCPULATION

(1) N	(2) M	(3) MULTIPLIER	(4) VAR OF EST/σ^2	(5) EFF(%)
19	6			79.75
19	7			78.21
19	8			75.77
19	9			72.55
19	10			72.55
19	11			64.62
19	12			60.33
19	13			56.05
19	14			51.89
19	15			47.93
19	16			44.20
19	17			40.73
19	18			37.49
19	19			34.46
20	1	-0.535482	0.079054	100.00
20	2			82.82
20	3			79.62
20	4			79.19
20	5			79.18
20	6			78.76
20	7			77.59
20	8			75.61
20	9			72.88
20	10			69.57
20	11			65.85
20	12			61.91
20	13			57.89
20	14			53.93
20	15			50.09
20	16			46.44
20	17			43.00
20	18			39.79
20	19			36.78
20	20			33.95

APPENDIX C

TABLE 3

An M-Order-Statistic Estimator
of the Parameter of a
1-Parameter Lognormal Population

TABLE 3

AN M-ORDER STATISTIC ESTIMATOR OF THE PARAMETER σ OF A 1 - PARAMETER LOG-NORMAL POPULATION

(1) N	(2) M	(3) K1	(4) K2	(5) K3	(6) K4	(7) K5	(8) VAR OF EST/σ^2
2	1	-1.77245					2.14159
2	2	-0.88623	0.88623				0.57080
3	1	-1.18164					0.78116
3	2	-1.18164	0.				0.78116
3	3	-0.59082	0.	0.59082			0.27548
4	1	-0.97146					0.46405
4	2	-1.08342	0.38802				0.42496
4	3	-0.72431	-0.18321	0.67337			0.28634
4	4	-0.45394	-0.11018	0.11018	0.45394		0.18005
5	1	-0.85987					0.33090
5	2	-0.92129	0.14430				0.32670
5	3	-0.92129	0.14430	0.			0.32670
5	4	-0.53069	-0.19686	0.	0.57649		0.18976
5	5	-0.37238	-0.13521	0.	0.13521	0.37238	0.13332
6	1	-0.78914					0.25901
6	2	-0.78426	-0.00963				0.25900
6	3	-0.70182	-0.31285	0.44716			0.23263
6	4	-0.55840	-0.24613	-0.08212	0.58492		0.18553
6	5	-0.42271	-0.18507	-0.05910	0.05384	0.50302	0.14063
6	6	-0.31752	-0.13856	-0.04321	0.04321	0.13856	
		0.31752					0.10570
7	1	-0.73955					0.21435
7	2	-0.67914	-0.10785				0.21249
7	3	-0.64365	-0.31820	0.31562			0.20189
7	4	-0.64365	-0.31820	0.31562	0.		0.20189
7	5	-0.44748	-0.21855	-0.10506	0.	0.54533	0.14080
7	6	-0.35330	-0.17205	-0.08064	0.	0.07604	
		0.44457					0.11124
7	7	-0.27781	-0.13510	-0.06246	0.	0.06246	
		0.13510	0.27781				0.08750
8	1	-0.70244					0.18400
8	2	-0.59891	-0.17294				0.17951

TABLE 3

AN M-ORDER STATISTIC ESTIMATOR
OF THE PARAMETER σ OF A
1 - PARAMETER LOG-NORMAL POPULATION

(1) N	(2) M	(3) K1	(4) K2	(5) K3	(6) K4	(7) K5	(8) VAR OF EST/σ^2
8	3	-0.58510	-0.30996	0.20538			0.17565
8	4	-0.53084	-0.27983	-0.15790	0.45137		0.15961
8	5	-0.45308	-0.23797	-0.13293	-0.04661	0.53916	0.13637
8	6	-0.37391	-0.19592	-0.10866	-0.03655	0.03109	
		0.50554					0.11262
8	7	-0.30468	-0.15941	-0.08802	-0.02878	0.02721	
		0.08554	0.39872				0.09180
8	8	-0.24759	-0.12945	-0.07131	-0.02296	0.02296	
		0.07131	0.12945	0.24759			0.07461
9	1	-0.67339					0.16205
9	2	-0.53667	-0.21778				0.15527
9	3	-0.53232	-0.29643	0.11691			0.15415
9	4	-0.50071	-0.27787	-0.17230	0.36853		0.14518
9	5	-0.50071	-0.27787	-0.17230	0.36853	0.	0.14518
9	6	-0.38247	-0.21125	-0.12953	-0.06538	0.	
		0.52105					0.11106
9	7	-0.32170	-0.17746	-0.10845	-0.05335	0.	
		0.04809	0.46671				0.09345
9	8	-0.26858	-0.14804	-0.09029	-0.04366	0.	
		0.04208	0.08834	0.36194			0.07804
9	9	-0.22373	-0.12327	-0.07510	-0.03597	0.	
		0.03597	0.07510	0.12327	0.22373		0.06502
10	1	-0.64988					0.14543
10	2	-0.48735	-0.24975				0.13686
10	3	-0.48659	-0.28147	0.04663			0.13669
10	4	-0.46878	-0.27051	-0.17843	0.29080		0.13182
10	5	-0.43115	-0.24826	-0.16304	-0.09569	0.44795	0.12133
10	6	-0.38207	-0.21965	-0.14376	-0.08358	-0.03008	
		0.51143					0.10757
10	7	-0.33068	-0.18989	-0.12398	-0.07157	-0.02478	
		0.01999	0.49842				0.09314

TABLE 3

AN M-ORDER STATISTIC ESTIMATOR OF THE PARAMETER σ OF A 1 - PARAMETER LOG-NORMAL POPULATION

(1) N	(2) M	(3) K1	(4) K2	(5) K3	(6) K4	(7) K5	(8) VAR OF EST/σ^2
10	8	-0.28276	-0.16225	-0.10575	-0.06075	-0.02044	
		0.01835	0.05793	0.43244			0.07966
10	9	-0.24063	-0.13801	-0.08986	-0.05145	-0.01697	
		0.01633	0.05057	0.08826	0.33180		0.06781
10	10	-0.20438	-0.11719	-0.07626	-0.04358	-0.01422	
		0.01422	0.04358	0.07626	0.11719	0.20438	0.05760
11	1	-0.63034					0.13241
11	2	-0.44747	-0.27320				0.12247
11	3	-0.44748	-0.26676	-0.00936			0.12247
11	4	-0.43789	-0.26060	-0.17960	0.22148		0.11993
11	5	-0.41257	-0.24512	-0.16842	-0.10855	0.39004	0.11307
11	6	-0.41257	-0.24512	-0.16842	-0.10855	0.39004	
		0.					0.11307
11	7	-0.33328	-0.19756	-0.13514	-0.08619	-0.04463	
		0.	0.50305				0.09142
11	8	-0.29129	-0.17255	-0.11787	-0.07491	-0.03784	
		0.	0.03290	0.47240			0.07992
11	9	-0.25258	-0.14955	-0.10206	-0.06471	-0.03209	
		0.	0.02989	0.06246	0.40258		0.06931
11	10	-0.21831	-0.12922	-0.08813	-0.05579	-0.02733	
		0.	0.02664	0.05507	0.08678	0.30663	0.05991
11	11	-0.18834	-0.11146	-0.07599	-0.04806	-0.02338	
		0.	0.02338	0.04806	0.07599	0.11146	
		0.18834					0.05169
12	1	-0.61[illegible]79					0.12193
12	2	-0.41462	-0.29083				0.11097
12	3	-0.41408	-0.25299	-0.05437			0.11077
12	4	-0.40927	-0.24976	-0.17789	0.16111		0.10954
12	5	-0.39254	-0.23924	-0.17001	-0.11653	0.33249	0.10512
12	6	-0.36504	-0.22223	-0.15763	-0.10760	-0.06433	
		0.44352					0.09780

TABLE 3

AN M-ORDER STATISTIC ESTIMATOR
OF THE PARAMETER σ OF A
1 - PARAMETER LOG-NORMAL POPULATION

(1) N	(2) M	(3) K1	(4) K2	(5) K3	(6) K4	(7) K5	(8) VAR OF EST/σ^2
12	7	-0.33109	-0.20140	-0.14263	-0.09705	-0.05752	
		-0.02104	0.49286				0.08874
12	8	-0.29516	-0.17943	-0.12693	-0.08615	-0.05071	
		-0.01790	0.01385	0.48979			0.07913
12	9	-0.26040	-0.15823	-0.11184	-0.07576	-0.04436	
		-0.01523	0.01308	0.04170	0.44721		0.06982
12	10	-0.22850	-0.13880	-0.09805	-0.06634	-0.03870	
		-0.01301	0.01204	0.03748	0.06447	0.37655	0.06127
12	11	-0.20003	-0.12148	-0.08578	-0.05799	-0.03374	
		-0.01118	0.01087	0.03335	0.05738	0.08462	
		0.28529					0.05364
12	12	-0.17480	-0.10615	-0.07494	-0.05063	-0.02942	
		-0.00966	0.00966	0.02942	0.05063	0.07494	
		0.10615	0.17480				0.04688
13	1	-0.59952					0.11329
13	2	-0.38713	-0.30433				0.10157
13	3	-0.38544	-0.24037	-0.09092			0.10105
13	4	-0.38331	-0.23886	-0.17454	0.10907		0.10053
13	5	-0.37248	-0.23187	-0.16916	-0.12110	0.27807	0.09773
13	6	-0.35235	-0.21915	-0.15964	-0.11395	-0.07480	
		0.40013					0.09249
13	7	-0.35235	-0.21915	-0.15964	-0.11395	-0.07480	
		0.40013	0.				0.09249
13	8	-0.29534	-0.18345	-0.13335	-0.09478	-0.06160	
		-0.03242	0.	0.48946			0.07757
13	9	-0.26474	-0.16437	-0.11939	-0.08474	-0.05488	
		-0.02822	0.	0.02382	0.47203		0.06954
13	10	-0.23557	-0.14622	-0.10615	-0.07526	-0.04862	
		-0.02454	0.	0.02218	0.04646	0.42373	0.06189

TABLE 3

AN M-ORDER STATISTIC ESTIMATOR
OF THE PARAMETER σ OF A
1 - PARAMETER LOG-NORMAL POPULATION

(1) N	(2) M	(3) K1	(4) K2	(5) K3	(6) K4	(7) K5	(8) VAR OF EST/σ^2
13	11	-0.208 2	-0.12959	-0.09404	-0.06662	-0.04296	
		-0.02138	0.	0.02030	0.04195	0.06504	
		0.35375					0.05486
13	12	-0.18477	-0.11465	-0.08318	-0.05889	-0.03793	
		-0.01870	0.	0.01835	0.03760	0.05836	
		0.08216	0.26695				0.04855
13	13	-0.16321	-0.10126	-0.07346	-0.05200	-0.03346	
		-0.01641	0.	0.01641	0.03346	0.05200	
		0.07346	0.10126	0.16321			0.04288
14	1	-0.58707					0.10606
14	2	-0.36379	-0.31486				0.09377
14	3	-0.36075	-0.22892	-0.12095			0.09289
14	4	-0.36001	-0.22836	-0.17034	0.06432		0.09272
14	5	-0.35317	-0.22384	-0.16677	-0.12334	0.22800	0.09099
14	6	-0.33862	-0.21446	-0.15959	-0.11778	-0.08221	
		0.35558					0.08727
14	7	-0.31765	-0.20105	-0.14947	-0.11010	-0.07657	
		-0.04626	0.43939				0.08189
14	8	-0.29270	-0.18517	-0.13755	-0.10118	-0.07014	
		-0.04204	-0.01555	0.47957			0.07548
14	9	-0.26619	-0.16833	-0.12497	-0.09181	-0.06350	
		-0.03781	-0.01354	0.01012	0.48168		0.06865
14	10	-0.23997	-0.15171	-0.11257	-0.08263	-0.05704	
		-0.03379	-0.01178	0.00974	0.03139	0.45342	0.06190
14	11	-0.21519	-0.13602	-0.10089	-0.07401	-0.05101	
		-0.03010	-0.01027	0.00916	0.02879	0.04920	
		0.40218					0.05551
14	12	-0.19243	-0.12161	-0.09018	-0.06612	-0.04552	
		-0.02678	-0.00899	0.00848	0.02617	0.04467	
		0.06474	0.33366				0.04964

TABLE 3

AN M-ORDER STATISTIC ESTIMATOR OF THE PARAMETER σ OF A 1 - PARAMETER LOG-NORMAL POPULATION

(1) N	(2) M	(3) K1	(4) K2	(5) K3	(6) K4	(7) K5	(8) VAR OF EST/σ^2
14	13	-0.17182	-0.10858	-0.08050	-0.05900	-0.04060	
		-0.02383	-0.00791	0.00774	0.02363	0.04031	
		0.05854	0.07959	0.25101			0.04433
14	14	-0.15316	-0.09678	-0.07175	-0.05258	-0.03616	
		-0.02121	-0.00699	0.00699	0.02121	0.03616	
		0.05258	0.07175	0.09678	0.15316		0.03951
15	1	-0.57607					0.09990
15	2	-0.34374	-0.32317				0.08719
15	3	-0.33930	-0.21857	-0.14586			0.08596
15	4	-0.33917	-0.21845	-0.16574	0.02582		0.08593
15	5	-0.33501	-0.21564	-0.16346	-0.12398	0.18263	0.08490
15	6	-0.32461	-0.20881	-0.15814	-0.11975	-0.08732	
		0.31192					0.08229
15	7	-0.30840	-0.19828	-0.15004	-0.11346	-0.08250	
		-0.05473	0.40527				0.07820
15	8	-0.30840	-0.19828	-0.15004	-0.11346	-0.08250	
		-0.05473	0.40527	0.			0.07820
15	9	-0.26535	-0.17046	-0.12883	-0.09722	-0.07040	
		-0.04627	-0.02464	0.	0.47892		0.06731
15	10	-0.24210	-0.15549	-0.11747	-0.08857	-0.06405	
		-0.04195	-0.02184	0.	0.01800	0.46936	0.06142
15	11	-0.21947	-0.14092	-0.10643	-0.08020	-0.05793	
		-0.03784	-0.01935	0.	0.01704	0.03587	
		0.43507					0.05568
15	12	-0.19817	-0.12723	-0.09606	-0.07236	-0.05222	
		-0.03404	-0.01716	0.	0.01589	0.03295	
		0.05067	0.38254				0.05028
15	13	-0.17856	-0.11463	-0.08653	-0.06516	-0.04699	
		-0.03059	-0.01525	0.	0.01466	0.03007	
		0.04625	0.06394	0.31585			0.04531

TABLE 3

AN M-ORDER STATISTIC ESTIMATOR
OF THE PARAMETER σ OF A
1 - PARAMETER LOG-NORMAL POPULATION

(1) N	(2) M	(3) K1	(4) K2	(5) K3	(6) K4	(7) K5	(8) VAR OF EST/σ^2
15	14	-0.16069	-0.10315	-0.07786	-0.05861	-0.04225	
		-0.02747	-0.01360	0.	0.01340	0.02730	
		0.04200	0.05819	0.07704	0.23701		0.04077
15	15	-0.14436	-0.09266	-0.06994	-0.05265	-0.03794	
		-0.02466	-0.01215	0.	0.01215	0.02466	
		0.03794	0.05265	0.06994	0.09266	0.14436	0.03663
16	1	-0.56625					0.09459
16	2	-0.32632	-0.32982				0.08157
16	3	-0.32055	-0.20922	-0.16675			0.08001
16	4	-0.32055	-0.20923	-0.16102	-0.00742		0.08001
16	5	-0.31814	-0.20756	-0.15963	-0.12355	0.14185	0.07943
16	6	-0.31081	-0.20267	-0.15576	-0.12041	-0.09069	
		0.27028					0.07762
16	7	-0.29838	-0.19448	-0.14936	-0.11533	-0.08670	
		-0.06116	0.36953				0.07453
16	8	-0.28187	-0.18364	-0.14096	-0.10874	-0.08160	
		-0.05737	-0.03490	0.43578			0.07042
16	9	-0.26271	-0.1711[illegible]	-0.13127	-0.10[illegible]19	-0.07583	
		-0.05315	-0.03209	-0.01197	0.46958		0.06564
16	10	-0.24235	-0.15781	-0.12102	-0.09323	-0.06978	
		-0.04879	-0.02928	-0.01060	0.00770	0.47452	0.06056
16	11	-0.22192	-0.14448	-0.11077	-0.08529	-0.06377	
		-0.04451	-0.02657	-0.00938	0.00751	0.02445	
		0.45549					0.05546
16	12	-0.20223	-0.13164	-0.10090	-0.07766	-0.05803	
		-0.04043	-0.02404	-0.00831	0.00717	0.02275	
		0.03877	0.41747				0.05055
16	13	-0.18374	-0.11959	-0.09165	-0.07052	-0.05266	
		-0.03665	-0.02172	-0.00738	0.00675	0.02100	
		0.03572	0.05131	0.36465			0.04593

TABLE 3

AN M-ORDER STATISTIC ESTIMATOR
OF THE PARAMETER σ OF A
1 - PARAMETER LOG-NORMAL POPULATION

(1) N	(2) M	(3) K1	(4) K2	(5) K3	(6) K4	(7) K5	(8) VAR OF EST/σ^2
16	14	-0.16666	-0.10846	-0.08311	-0.06393	-0.04773	
		-0.03319	-0.01962	-0.00658	0.00628	0.01928	
		0.03274	0.04707	0.06285	0.29996		0.04166
16	15	-0.15100	-0.09826	-0.07529	-0.05791	-0.04322	
		-0.03003	-0.01773	-0.00589	0.00579	0.01761	
		0.02988	0.04299	0.05753	0.07455	0.22461	0.03774
16	16	-0.13659	-0.08888	-0.06810	-0.05237	-0.03908	
		-0.02715	-0.01601	-0.00529	0.00529	0.01601	
		0.02715	0.03908	0.05237	0.06810	0.08888	
		0.13659					0.03414
17	1	-0.55743					0.08947
17	2	-0.31104	-0.33517				0.07672
17	3	-0.30403	-0.20075	-0.18441			0.07487
17	4	-0.30387	-0.20070	-0.15634	-0.03623		0.07482
17	5	-0.30259	-0.19978	-0.15555	-0.12241	0.10534	0.07452
17	6	-0.29750	-0.19634	-0.15278	-0.12012	-0.09279	
		0.23126					0.07328
17	7	-0.28806	-0.19003	-0.14779	-0.11609	-0.08954	
		-0.06599	0.33367				0.07097
17	8	-0.27479	-0.18121	-0.14086	-0.11056	-0.08516	
		-0.06260	-0.04181	0.40798			0.06771
17	9	-0.27479	-0.18121	-0.14086	-0.11056	-0.08516	
		-0.06260	-0.04181	0.40798	0.		0.06771
17	10	-0.24107	-0.15889	-0.12342	-0.09675	-0.07437	
		-0.05445	-0.03605	-0.01935	0.	0.47054	0.05942
17	11	-0.22283	-0.14685	-0.11403	-0.08935	-0.06863	
		-0.05018	-0.03311	-0.01741	0.	0.01405	
		0.46590					0.05493
17	12	-0.20482	-0.13495	-0.10478	-0.08207	-0.06300	
		-0.04600	-0.03027	-0.01564	0.	0.01346	
		0.02851	0.44110				0.05049

TABLE 3

AN M-ORDER STATISTIC ESTIMATOR
OF THE PARAMETER σ OF A
1 - PARAMETER LOG-NORMAL POPULATION

(1) N	(2) M	(3) K1	(4) K2	(5) K3	(6) K4	(7) K5	(8) VAR OF EST/σ^2
17	13	-0.18755	-0.12356	-0.09592	-0.07511	-0.05763	
		-0.04204	-0.02761	-0.01406	0.	0.01273	
		0.02652	0.04060	0.40086			0.04624
17	14	-0.17135	-0.11288	-0.08761	-0.06859	-0.05261	
		-0.03835	-0.02514	-0.01266	0.	0.01191	
		0.02453	0.03753	0.05140	0.34835		0.04224
17	15	-0.15633	-0.10298	-0.07992	-0.06256	-0.04797	
		-0.03495	-0.02288	-0.01143	0.	0.01107	
		0.02259	0.03455	0.04737	0.06158	0.28570	0.03854
17	16	-0.14248	-0.09385	-0.07284	-0.05701	-0.04370	
		-0.03183	-0.02082	-0.01033	0.	0.01021	
		0.02072	0.03169	0.04350	0.05666	0.07216	
		0.21355					0.03513
17	17	-0.12966	-0.08541	-0.06628	-0.05187	-0.03976	
		-0.02895	-0.01893	-0.00936	0.	0.00936	
		0.01893	0.02895	0.03976	0.05187	0.06628	
		0.08541	0.12966				0.03197
18	1	-0.54944					0.08590
18	2	-0.29752	-0.33953				0.07249
18	3	-0.28938	-0.19307	-0.19948			0.07038
18	4	-0.28891	-0.19283	-0.15179	-0.06132		0.07025
18	5	-0.28831	-0.19238	-0.15139	-0.12080	0.07270	0.07012
18	6	-0.28486	-0.19001	-0.14946	-0.11917	-0.09393	
		0.19508					0.06929
18	7	-0.27775	-0.18520	-0.14561	-0.11601	-0.09134	
		-0.06954	0.29864				0.06757
18	8	-0.26715	-0.17808	-0.13995	-0.11143	-0.08764	
		-0.06660	-0.04730	0.37850			0.06500
18	9	-0.25380	-0.16913	-0.13287	-0.10574	-0.08308	
		-0.06303	-0.04463	-0.02727	0.43268		0.06176
18	10	-0.23860	-0.15897	-0.12485	-0.09931	-0.07797	
		-0.05907	-0.04171	-0.02531	-0.00950	0.46181	0.05807

TABLE 3

AN M-ORDER STATISTIC ESTIMATOR
OF THE PARAMETER σ OF A
1 - PARAMETER LOG-NORMAL POPULATION

(1) N	(2) M	(3) K1	(4) K2	(5) K3	(6) K4	(7) K5	(8) VAR OF EST/σ^2
18	11	-0.22246	-0.14819	-0.11636	-0.09252	-0.07259	
		-0.05493	-0.03869	-0.02334	-0.00852	0.00605	
		0.46831					0.05415
18	12	-0.20613	-0.13729	-0.10778	-0.08567	-0.06718	
		-0.05079	-0.03571	-0.02144	-0.00764	0.00595	
		0.01956	0.45547				0.05018
18	13	-0.19016	-0.12664	-0.09940	-0.07899	-0.06192	
		-0.04678	-0.03283	-0.01963	-0.00685	0.00575	
		0.01840	0.03132	0.42677			0.04629
18	14	-0.17491	-0.11648	-0.09141	-0.07263	-0.05691	
		-0.04297	-0.03012	-0.01795	-0.00616	0.00548	
		0.01719	0.02918	0.04170	0.38529		0.04258
18	15	-0.16059	-0.10693	-0.08391	-0.06666	-0.05222	
		-0.03941	-0.02760	-0.01641	-0.00556	0.00517	
		0.01598	0.02707	0.03868	0.05114	0.33347	0.03909
18	16	-0.14727	-0.09806	-0.07695	-0.06112	-0.04787	
		-0.03611	-0.02527	-0.01499	-0.00502	0.00484	
		0.01478	0.02501	0.03576	0.04734	0.06023	
		0.27283					0.03585
18	17	-0.13494	-0.08985	-0.07050	-0.05599	-0.04385	
		-0.03307	-0.02313	-0.01371	-0.00456	0.00450	
		0.01363	0.02304	0.03295	0.04366	0.05568	
		0.06988	0.20361				0.03285
18	18	-0.12345	-0.08220	-0.06449	-0.05122	-0.04011	
		-0.03025	-0.02115	-0.01252	-0.00415	0.00415	
		0.01252	0.02115	0.03025	0.04011	0.05122	
		0.06449	0.08220	0.12345			0.03006
19	1	-0.54216					0.08228
19	2	-0.28548	-0.34308				0.06877
19	3	-0.27632	-0.18607	-0.21243			0.06644
19	4	-0.27545	-0.18558	-0.14743	-0.08328		0.06621
19	5	-0.27523	-0.18541	-0.14727	-0.11890	0.04351	0.06616
19	6	-0.27295	-0.18382	-0.14595	-0.11777	-0.09437	
		0.16175					0.06562

TABLE 3

AN M-ORDER STATISTIC ESTIMATOR OF THE PARAMETER σ OF A 1 - PARAMETER LOG-NORMAL POPULATION

(1) N	(2) M	(3) K1	(4) K2	(5) K3	(6) K4	(7) K5	(8) VAR OF EST/σ^2
19	7	-0.26764	-0.18019	-0.14301	-0.11533	-0.09233	
		-0.07210	0.26501				0.06435
19	8	-0.25924	-0.17448	-0.13843	-0.11158	-0.08925	
		-0.06959	-0.05164	0.34843			0.06234
19	9	-0.24819	-0.16701	-0.13246	-0.10672	-0.08530	
		-0.06642	-0.04917	-0.03299	0.40941		0.05969
19	10	-0.24819	-0.16701	-0.13246	-0.10672	-0.08530	
		-0.06642	-0.04917	-0.03299	0.40941	0.	0.05969
19	11	-0.22103	-0.14868	-0.11787	-0.09489	-0.07575	
		-0.05887	-0.04342	-0.02889	-0.01562	0.	
		0.46373					0.05317
19	12	-0.20635	-0.13878	-0.11000	-0.08853	-0.07065	
		-0.05486	-0.04040	-0.02680	-0.01420	0.	
		0.01126	0.46228				0.04964
19	13	-0.19170	-0.12891	-0.10217	-0.08220	-0.06557	
		-0.05089	-0.03743	-0.02476	-0.01290	0.	
		0.01088	0.02318	0.44414			0.04612
19	14	-0.17746	-0.11933	-0.09456	-0.07607	-0.06066	
		-0.04705	-0.03458	-0.02282	-0.01173	0.	
		0.01039	0.02178	0.03326	0.41282		0.04270
19	15	-0.16391	-0.11021	-0.08732	-0.07024	-0.05600	
		-0.04342	-0.03188	-0.02100	-0.01067	0.	
		0.00984	0.02035	0.03106	0.04229	0.37077	0.03944
19	16	-0.15116	-0.10163	-0.08052	-0.06476	-0.05162	
		-0.04001	-0.02936	-0.01932	-0.00972	0.	
		0.00925	0.01895	0.02889	0.03936	0.05065	
		0.31985					0.03637
19	17	-0.13927	-0.09364	-0.07418	-0.05966	-0.04755	
		-0.03684	-0.02702	-0.01776	-0.00887	0.	
		0.00864	0.01758	0.02679	0.03652	0.04706	
		0.05884	0.26115				0.03351

TABLE 3

AN M-ORDER STATISTIC ESTIMATOR OF THE PARAMETER σ OF A 1 - PARAMETER LOG-NORMAL POPULATION

(1) N	(2) M	(3) K1	(4) K2	(5) K3	(6) K4	(7) K5	(8) VAR OF EST/σ^2
19	18	-0.12821	-0.08620	-0.06829	-0.05491	-0.04376	
		-0.03390	-0.02486	-0.01633	-0.00812	0.	
		0.00804	0.01626	0.02478	0.03379	0.04359	
		0.05462	0.06771	0.19463			0.03085
19	19	-0.11785	-0.07923	-0.06277	-0.05047	-0.04022	
		-0.03116	-0.02284	-0.01500	-0.00743	0.	
		0.00743	0.01500	0.02284	0.03116	0.04022	
		0.05047	0.06277	0.07923	0.11785		0.02836
20	1	-0.53548					0.07905
20	2	-0.27468	-0.34601				0.06547
20	3	-0.26461	-0.17969	-0.22364			0.06294
20	4	-0.26328	-0.17889	-0.14327	-0.10259		0.06260
20	5	-0.26324	-0.17886	-0.14324	-0.11682	0.01737	0.06260
20	6	-0.26179	-0.17783	-0.14237	-0.11607	-0.09430	
		0.13116					0.06226
20	7	-0.25787	-0.17513	-0.14016	-0.11421	-0.09272	
		-0.07388	0.23311				0.06134
20	8	-0.25125	-0.17059	-0.13646	-0.11116	-0.09019	
		-0.07178	-0.05503	0.31855			0.05977
20	9	-0.24217	-0.16438	-0.13148	-0.10705	-0.08679	
		-0.06901	-0.05281	-0.03769	0.38453		0.05761
20	10	-0.23113	-0.15687	-0.12544	-0.10209	-0.08273	
		-0.06572	-0.05022	-0.03573	-0.02191	0.43002	0.05500
20	11	-0.21877	-0.14845	-0.11869	-0.09657	-0.07822	
		-0.06208	-0.04738	-0.03362	-0.02048	-0.00773	
		0.45558					0.05206
20	12	-0.20566	-0.13954	-0.11154	-0.09073	-0.07346	
		-0.05827	-0.04442	-0.03145	-0.01905	-0.00701	
		0.00487	0.46294				0.04894

TABLE 3

AN M-ORDER STATISTIC ESTIMATOR
OF THE PARAMETER σ OF A
1 - PARAMETER LOG-NORMAL POPULATION

(1) N	(2) M	(3) K1	(4) K2	(5) K3	(6) K4	(7) K5	(8) VAR OF EST/σ^2
20	13	-0.19231	-0.13047	-0.10428	-0.08481	-0.06864	
		-0.05442	-0.04144	-0.02928	-0.01766	-0.00635	
		0.00482	0.01599	0.45440			0.04577
20	14	-0.17913	-0.12151	-0.09711	-0.07896	-0.06389	
		-0.05063	-0.03853	-0.02718	-0.01632	-0.00575	
		0.00470	0.01517	0.02582	0.43245		0.04263
20	15	-0.16638	-0.11286	-0.09019	-0.07332	-0.05932	
		-0.04699	-0.03573	-0.02518	-0.01507	-0.00522	
		0.00452	0.01431	0.02427	0.03457	0.39941	0.03960
20	16	-0.15425	-0.10462	-0.08360	-0.06796	-0.05497	
		-0.04353	-0.03309	-0.02329	-0.01390	-0.00475	
		0.00431	0.01342	0.02272	0.03235	0.04252	
		0.35723					0.03671
20	17	-0.14283	-0.09687	-0.07740	-0.06292	-0.05088	
		-0.04029	-0.03061	-0.02152	-0.01282	-0.00433	
		0.00408	0.01255	0.02120	0.03018	0.03970	
		0.04999	0.30734				0.03399
20	18	-0.13214	-0.08962	-0.07161	-0.05820	-0.04706	
		-0.03726	-0.02830	-0.01989	-0.01183	-0.00396	
		0.00384	0.01169	0.01973	0.02809	0.03696	
		0.04661	0.05745	0.25050			0.03145
20	19	-0.12216	-0.08285	-0.06619	-0.05380	-0.04350	
		-0.03443	-0.02615	-0.01837	-0.01091	-0.00363	
		0.00359	0.01086	0.01831	0.02607	0.03433	
		0.04334	0.05353	0.06566	0.18647		0.02908
20	20	-0.11276	-0.07648	-0.06110	-0.04966	-0.04015	
		-0.03178	-0.02413	-0.01695	-0.01006	-0.00334	
		0.00334	0.01006	0.01695	0.02413	0.03178	
		0.04015	0.04966	0.06110	0.07648	0.11276	0.02684

Vita

Eugene Raymond Highfield was born [redacted] the son of [redacted] and [redacted] After completing [redacted] he enrolled at Michigan State University, East Lansing. He received his commission as a 2nd Lt. upon graduation in June 1952 and entered the Air Force at Ellington AFB, Texas, where he was a student in basic navigation. Finishing his navigation training at Mather AFB, California, in November 1953, he spent the next two and a half years on a B-36 crew at Loring AFB, Maine. In August 1956 he entered a one year Aeronautical Engineering curriculum at AFIT and then went overseas for three years in an aircraft maintenance assignment. Upon his return from overseas he was assigned in aircraft maintenance at Maxwell AFB, Alabama, where he spent two years before arriving at the Air Force Institute of Technology to study Reliability Engineering. Capt. Highfield is a member of the Institute of Electronic and Electrical Engineers and Tau Beta Pi.

Permanent address: [redacted]

This thesis was typed by Mrs. Shirley McCarthy.